FYUG Odd Semester Exam., 2024

MATHEMATICS

(3rd Semester)

Course No. : MATDSM–201T

(Classical Algebra and Trigonometry)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) Find the adjoint of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$$

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(b) Define rank of a matrix.

(c) Find the inverse of the matrix

$$A=\begin{bmatrix}5 & 2\\ 7 & 3\end{bmatrix}$$

2. (a) Find the rank of the matrix

$$A=\begin{bmatrix}1 & -1 & 3 & 6\\ 1 & 3 & -3 & -4\\ 5 & 3 & 3 & 11\end{bmatrix}$$

by reducing it to echelon form. 5

(b) Solve the following system of equations by Gaussian elimination method : 5

$$5x-2y+z=4$$
$$7x+y-5z=8$$
$$3x+7y+4z=10$$

OR

3. (a) Find the inverse of the matrix

$$A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$$ 5

(b) If A and B are invertible matrices of the same order, then show that AB is invertible and $(AB)^{-1}=B^{-1}A^{-1}$. 5

UNIT—II

4. Answer any *two* from the following : 2×2=4

(a) If α, β and γ are the roots of the equation $x^3+px^2+qx+r=0$, then find the value of $\Sigma\alpha^2\beta$.

(b) Find the nature of the roots of the equation $x^4+2x^2+3x-1=0$.

(c) Remove the fractional coefficient of the equation

$$x^4-\frac{2}{3}x^3-\frac{5}{6}x^2+\frac{7}{72}x+\frac{11}{216}=0$$

5. (a) If α, β and γ be the roots of the equation $x^3+px+r=0$, then find the value of

$$\sum\frac{1}{\alpha^2-\beta\gamma}$$ 5

(b) Solve the following equation by Cardan's method : 5

$$x^3-6x-4=0$$

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OR

6. *(a)* Solve the equation

$$4x^3 + 20x^2 - 23x + 6 = 0$$

two of its roots being equal. 5

(b) Solve $x^4 + x^3 - 16x^2 - 4x + 48 = 0$, if product of its roots is 8. 5

UNIT—III

7. Answer any *two* from the following : 2×2=4

(a) Write the statement of Cauchy-Schwarz inequality.

(b) If x be a positive number, then show that

$$x + \frac{1}{x} \geq 2$$

(c) Prove that the arithmetic mean of two positive numbers is always greater than or equal to their geometric mean.

8. *(a)* If a, b and c be positive, then show that

$$b^2c^2 + c^2a^2 + a^2b^2 \geq abc(a + b + c)$$ 5

(b) Prove Cauchy-Schwarz inequality. 5

OR

9. *(a)* If x, y and z are positive and $x + y + z = 1$, then show that

$$(1 - x)(1 - y)(1 - z) \geq 8xyz$$ 5

(b) If a, b and c be positive, then prove that

$$\frac{b^2 + c^2}{b + c} + \frac{c^2 + a^2}{c + a} + \frac{a^2 + b^2}{a + b} \geq a + b + c$$ 5

UNIT—IV

10. Answer any *two* from the following : 2×2=4

(a) State De Moivre's theorem for rational indices.

(b) Write down the polar form of the complex number $-1 + i$.

(c) Expand $\cos^2\theta$ in the powers of θ.

11. *(a)* Prove De Moivre's theorem for positive integral index. 5

(b) If $2\cos\theta = x + \frac{1}{x}$ and θ is real, then prove that

$$2\cos n\theta = x^n + \frac{1}{x^n}$$

n being an integer. 5

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OR

12. *(a)* Expand $\cos^7\theta$ in a series of cosines of multiples of θ. 5

(b) If n be an integer, then prove that

$$\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2}-n\theta\right) + i\sin\left(\frac{n\pi}{2}-n\theta\right) \quad 5$$

UNIT—V

13. Answer any *two* from the following : 2×2=4

(a) Show that

$$\frac{\pi}{12} = \left(1-\frac{1}{3^{\frac{1}{2}}}\right) - \frac{1}{3}\left(1-\frac{1}{3^{\frac{3}{2}}}\right) + \frac{1}{5}\left(1-\frac{1}{3^{\frac{5}{2}}}\right) - \cdots\infty$$

(b) Find all values of z such that $\sin z = 0$.

(c) Show that $\frac{\pi}{8} = \frac{1}{1\cdot3} + \frac{1}{5\cdot7} + \frac{1}{9\cdot11} + \cdots\infty$.

14. *(a)* State and prove Gregory's series. 5

(b) Find the sum of the series

$$\cos\theta + \frac{\operatorname{cosec}\theta}{\lfloor 1}\cdot\cos 2\theta + \frac{\operatorname{cosec}^2\theta}{\lfloor 2}\cdot\cos 3\theta + \cdots \quad 5$$

OR

15. *(a)* Prove that

$$\log(x+iy) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{y}{x}\right) \quad 5$$

(b) If θ lies between 0 and $\frac{\pi}{2}$, then prove that

$$\tan^{-1}\left(\frac{1-\cos\theta}{1+\cos\theta}\right) = \tan^2\frac{\theta}{2} - \frac{1}{3}\tan^6\frac{\theta}{2} + \frac{1}{5}\tan^{10}\frac{\theta}{2} - \cdots \quad 5$$

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