FYUG Odd Semester Exam., 2024

MATHEMATICS

(3rd Semester)

Course No. : MATDSC-202T

(Ordinary Differential Equations)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) Define ordinary differential equation. Give an example of a nonlinear differential equation.

(b) Form the differential equation of the family of curves $y = a\cos nt + b\sin nt$.

(c) Solve $3e^x \tan y dx + (1 - e^x)\sec^2 y dy = 0$.

Scanned with OKEN Scanner

Answer any *one* from the following : 10

2. *(a)* Solve $(x^2 - 2y^2)dy + 2xydx = 0$. 4

(b) Solve $(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$. 3

(c) Show that $xdx + ydy + \frac{xdy - ydx}{x^2+y^2} = 0$ is exact and hence find the solution. 3

3. *(a)* Prove that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ 5

(b) Solve $(2x + y - 3)dy = (x + 2y - 3)dx$. 5

UNIT—II

4. Answer any *two* from the following : 2×2=4

(a) Define integrating factor. Find the integrating factor from the differential equation

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$

(b) Write the method of solving linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are constants or function of x only.

(Continued)

(c) Write the method of solving of the equation $y = xp + f(p)$, $P = \frac{dy}{dx}$.

Answer any *one* from the following : 10

5. *(a)* If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ is a function of x, say $f(x)$, then prove that $e^{\int f(x)dx}$ is the integrating factor of $Mdx + Ndy = 0$. 4

(b) Solve $(1+y^2)dx = (\tan^{-1}y - x)dy$. 3

(c) Solve $p^2 + 2px + py + 2xy = 0$, where $p = \frac{dy}{dx}$. 3

6. *(a)* Solve $x^2ydx - (x^3 + y^3)dy = 0$. 3

(b) Solve $x\frac{dy}{dx} + y = y^2 \log x$. 4

(c) Reduce the differential equation $(px - y)(x - yp) = 2p$ to Clairaut's form by the substitution $x^2 = u$, $y^2 = v$ and find its complete solution. 3

(Turn Over)

Scanned with OKEN Scanner

UNIT—III

7. Answer any *two* from the following : 2×2=4

(a) Define homogeneous and non-homogeneous linear differential equation of order n.

(b) Define Wronskian of n functions. Also show that the functions e^{-x}, e^{x}, e^{2x} are linearly independent function.

(c) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2 = 0$ with $y = 0$, $x = 0$ and $\frac{dy}{dx} = 0$.

Answer any *one* from the following : 10

8. *(a)* Solve $(D^2 - 3D + 2)y = 3\sin 2x$, $D = \frac{d}{dx}$. 4

(b) Solve $(D^2 - 2D + 1)y = e^x x^2$, $D = \frac{d}{dx}$. 3

(c) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x\cos x$. 3

9. *(a)* Consider the linear differential equation $y''(x) + Py'(x) + Qy(x) = 0$, where P, Q are either constant or function of x alone. Then show that two solutions of the differential equation are linearly dependent iff their Wronskian is zero. 6

(b) Solve $(D^3 + 1)y = \cos 2x$, $D = \frac{d}{dx}$. 4

UNIT—IV

10. Answer any *two* from the following : 2×2=4

(a) Define orthogonal trajectory and isogonal trajectory.

(b) Reduce the equation

$$(x^2D^2 - xD + 2)y = x\log x, \quad D = \frac{d}{dx}$$

into a linear differential equation with constant coefficients.

(c) Define the method of undetermined coefficients for solving linear differential equation.

Answer any *one* from the following : 10

11. *(a)* Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 12e^{2x}\sin 3x$ by the method of undetermined coefficients. 5

(b) Solve $(x^3D^3 + 3x^2D^2 + xD + 1)y = x + \log x$, $D = \frac{d}{dx}$. 5

Scanned with OKEN Scanner

12. *(a)* Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \sin e^{-x}$ by the method of variation of parameters. 5

(b) Find the orthogonal trajectory of $r = c(1 - \cos\theta)$. 5

UNIT—V

13. Answer any *two* from the following : 2×2=4

(a) State necessary condition for the integrability of the total differential equation $Pdx + Qdy + Rdz = 0$, where P, Q, R are functions of x, y, z.

(b) Solve $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{xy^2}$.

(c) Solve $xdx + zdy + (y + 2z)dz = 0$.

Answer any *one* from the following : 10

14. *(a)* Solve $\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$. 5

(b) Solve $\frac{dx}{dt} = 4x - 2y$, $\frac{dy}{dt} = x + y$. 5

15. *(a)* Solve $(y^2 + z^2 - x^2)dx - 2xydy - 2xzdz = 0$. 5

(b) Solve $\frac{dx}{y - zx} = \frac{dy}{yz + x} = \frac{dz}{x^2 + y^2}$. 5

★ ★ ★

Scanned with OKEN Scanner