2024/FYUG/ODD/SEM/
MATDSC–201T/283

FYUG Odd Semester Exam., 2024

MATHEMATICS

(3rd Semester)

Course No. : MATDSC–201T

(Real Analysis)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) State completeness property of $\mathbb{R}$.

(b) If $a, b \in \mathbb{R}$, then prove that—

$$|a+b| \leq |a| + |b|$$

(c) Let

$$S = \left\{1 + \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$$

Find inf$S$ and sup$S$.

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2. *(a)* Prove that—

(i) if $a \in \mathbb{R}$ and $a \neq 0$, then $a^2 > 0$;

(ii) $1 > 0$;

(iii) if $n \in \mathbb{N}$, then $n > 0$. 6

(b) State and prove Archimedean property. 4

OR

3. *(a)* State and prove density theorem. 5

(b) If S is a subset of $\mathbb{R}$ that contains at least two points and has the property, if $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$, then show that S is an interval. 5

UNIT—II

4. Answer any *two* from the following : 2×2=4

(a) Prove that—

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

(b) Give an example of two divergent sequences X and Y such that *(i)* their sum $X + Y$ converges and *(ii)* their product XY converges.

(c) Define bounded sequence. Give an example of a bounded sequence which is not convergent.

5. *(a)* Prove that a sequence in $\mathbb{R}$ can have at most one limit. 4

(b) Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers that converge to x and y respectively. Then prove that the sequences $X + Y$ and XY converge to $x + y$ and xy respectively. 6

OR

6. *(a)* Suppose that $X = (x_n), Y = (y_n)$ and $Z = (z_n)$ are sequences of real numbers such that $x_n \leq y_n \leq z_n$, for all $n \in \mathbb{N}$ and that $\lim_{n \to \infty} (x_n) = \lim_{n \to \infty} (z_n)$. Then prove that $Y = (y_n)$ is convergent and

$$\lim_{n \to \infty} (x_n) = \lim_{n \to \infty} (y_n) = \lim_{n \to \infty} (z_n)$$ 5

(b) Prove that a monotone sequence of real numbers is convergent if it is bounded. 5

UNIT—III

7. Answer any *two* from the following : 2×2=4

(a) Define subsequence of a sequence. Give an example of it.

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(b) Prove that a Cauchy sequence of real numbers is bounded.

(c) Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$, for all $n \in \mathbb{N}$. If $\lim_{n\to\infty}(x_n) = +\infty$ then prove that—

$$\lim_{n\to\infty}(y_n) = +\infty$$

8. (a) If a sequence $X = (x_n)$ of real numbers converge to a real number x, then prove that any subsequence $X' = (x_{n_k})$ of X also converge to x. 5

(b) (i) State divergence criteria.

(ii) Prove that the sequence $X = \{(-1)^n\}$ is divergent. 2+3=5

OR

9. (a) If $X = (x_n)$ is a sequence of real numbers, then prove that there is a subsequence of X that is monotone. 6

(b) Prove that a Cauchy sequence of real numbers is bounded. 4

UNIT—IV

10. Answer any *two* from the following : 2×2=4

(a) Define cluster point of a set. Find the cluster points of the set

$$A = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$$

(b) Prove that—

$$\lim_{x\to 0} x^{3/2} = 0 \quad (x > 0)$$

(Use Squeeze theorem)

(c) Give an example of a function which is defined on $\mathbb{R}$ but is not continuous at any point of $\mathbb{R}$.

11. (a) Prove that a number $c \in \mathbb{R}$ is a cluster point of a subset A of $\mathbb{R}$ if and only if there exists a sequence (a_n) in A such that $\lim_{n\to\infty}(a_n) = c$ and $a_n \neq c$ for all $n \in \mathbb{N}$. 5

(b) If $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ has a limit at $c \in \mathbb{R}$, then prove that f is bounded on some neighbourhood of c. 5

OR

12. (a) Let $k > 0$ and let $f : \mathbb{R} \to \mathbb{R}$ satisfy the condition $|f(x) - f(y)| \leq k|x - y|$ for all $x, y \in \mathbb{R}$. Show that f is continuous at every point $c \in \mathbb{R}$. 5

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(b) Let $A, B \subseteq \mathbb{R}$, let $f : A \to \mathbb{R}$ be continuous on A, and let $g : B \to \mathbb{R}$ be continuous on B. If $f(A) \subseteq B$, then prove that the composite function $g \circ f : A \to B$ is continuous on A. 5

UNIT—V

13. Answer any *two* from the following : 2×2=4

(a) State maximum-minimum theorem.

(b) State Caratheodory's theorem.

(c) State extremum theorem.

14. (a) Let $I = [a, b]$ be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then prove that f is bounded on I. 6

(b) Let c be an interior point of the interval I at which $f : I \to \mathbb{R}$ has a relative extremum. If the derivative of f at c exists, then prove that $f'(c) = 0$. 4

OR

15. (a) Let $I = [a, b]$ and let $f : I \to \mathbb{R}$ be continuous on I. If $f(a) < 0 < f(b)$ or if $f(a) > 0 > f(b)$, then prove that there exists a number $c \in (a, b)$ such that $f(c) = 0$. 6

(Continued)

(b) Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then prove that the set $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval. 4

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