FYUG Even Semester Exam., 2026

MATHEMATICS

(4th Semester)

Paper Code : MTMDSC 252T

(Mechanics—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions.

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) State a set of necessary and sufficient conditions for the equilibrium of a system of coplanar forces acting on a rigid body.

(b) Write the statement of Lami's theorem and also converse of Lami's theorem.

(c) Define moment and couple of a force.

Scanned with OKEN Scanner

2. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Prove Lami's theorem. 5

(b) Forces P, Q, R and S act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ of a cyclic quadrilateral $ABCD$, taken in order, where A and B are the extremities of a diameter. If they are in equilibrium, then prove that

$$R^2 = P^2 + Q^2 + S^2 + \frac{2PQS}{R}$$ 5

(c) A uniform beam of length $2a$ rests in equilibrium against a smooth vertical wall and upon a peg at a distance b from the wall. Show that the inclination of the beam to the vertical is

$$\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$$ 5

(d) A beam whose centre of gravity divides it into two portions a and b is placed inside a smooth sphere. Show that if θ be its inclination to the horizontal in the position of equilibrium and 2α be the angle subtended by the beam at the centre of the sphere, then

$$\tan\theta = \left(\frac{b-a}{b+a}\right)\tan\alpha$$ 5

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Write any two laws of limiting friction.

(b) Define coefficient of friction and angle of friction, and establish the relation between them.

(c) Define CG of a body.

4. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) A uniform ladder of length l and weight W rests with its one end on a rough horizontal plane and its upper end against a smooth vertical wall, the inclination to the vertical being α. A force of magnitude P is applied horizontally to the ladder at a point distance c from its lower end so as to make the lower foot approach the wall. Prove that P must exceed

$$\frac{lW}{l-c}\left(\mu + \frac{1}{2}\tan\alpha\right)$$

where μ is the coefficient of friction at the foot. 5

Scanned with OKEN Scanner

(b) What height can a particle attain, being at rest inside a hollow sphere of radius a if coefficient of friction is $\frac{1}{\sqrt{3}}$? Also prove that the greatest distance from the vertical diameter is $\frac{a\mu}{\sqrt{1+\mu^2}}$, where μ is the coefficient of friction. 5

(c) Find the CG of the area bounded by the parabola $y^2 = 4ax$, the x-axis and the latus rectum. 5

(d) Find the CG of a uniform circular wire of radius a in the form of a quadrant of a circle. 5

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) The velocities of a particle along and perpendicular to the radius vector are respectively λ_r and μ_θ from a fixed origin. Find the path.

(b) Define SHM. Write also the expression for time period of SHM.

(c) If the tangential and normal accelerations of a particle describing a plane curve be constant throughout, then prove that the radius of curvature at any point is given by $\rho = (at + b)^2$.

6. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Derive expressions for tangential and normal components of acceleration. 6

(b) A point moves in a plane curve, so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve. 4

(c) A particle is performing an SHM of period T about a centre O and it passes through a point P, where $OP = b$ with velocity v in the direction OP. Prove that the time which elapses before it returns to P is

$$\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$$ 5

(d) A point moves in a plane curve so that its tangential acceleration is constant, and the magnitude of the tangential velocity and normal acceleration are in a constant ratio. Show that the intrinsic equation of the curve is of the form

$$S = A\psi^2 + B\psi + C$$ 5

Scanned with OKEN Scanner

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Find the expression for v^2, if the law of force is that of inverse square, v being the velocity at a distance x from the centre when the particle starts from rest at a distance a from the centre.

(b) Write the expressions for horizontal range, maximum horizontal range, time of flight for a projectile with velocity of projection u and angle of projection α.

(c) Write the equation of motion, when—

(i) a particle of mass m falls from the highest point of a vertical circle of radius a;

(ii) a particle is projected from the lowest point of a vertical circle of radius a with a velocity u.

8. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) A particle of mass m is acted on by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, then show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 6

(b) If v_1 and v_2 be the velocities at the ends of a focal chord of a projectile's path and v stands for the horizontal component of the velocity, then prove that

$$\frac{1}{v_1^2}+\frac{1}{v_2^2}=\frac{1}{v^2}$$ 4

(c) Show that the path of a projectile in vacuum is a parabola. Find its vertex. 5

(d) A particle slides down the outside of a smooth vertical circle starting from rest at the highest point. Find the expressions for velocity and normal reaction. 5

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Prove that impulse = change in momentum.

(b) A sphere impinges directly on an equal sphere at rest. If the coefficient of restitution be e, then prove that their velocities after impact are in the ratio

$$(1-e):(1+e)$$

(c) State Newton's experimental law for direct impact.

Scanned with OKEN Scanner

10. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) State and prove the principle of linear momentum. 5

(b) Three perfectly elastic balls of masses m_1, m_2 and m_3 are placed in a straight line. The first impinges directly on the second with a velocity u and the second impinges the third. Show that the third ball will move after impact with velocity u, if

$$(m_1 + m_2)(m_2 + m_3) = 4m_1m_2$$ 5

(c) Find the loss of kinetic energy by direct impact of two smooth spheres. 5

(d) A body of mass 10 kg and velocity 9 m/sec strikes another body of mass 20 kg and velocity 6 m/sec moving in the same direction. If the velocity of the first body becomes 6 m/sec after impact, then find the coefficient of restitution and loss of kinetic energy. 5

★ ★ ★

Scanned with OKEN Scanner