FYUG Even Semester Exam., 2026

MATHEMATICS

(6th Semester)

Paper Code : MTMDSC 353T

(Multivariate Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions.

UNIT—1

1. Answer any *two* of the following questions : 2×2=4

(a) Show that $\lim_{(x,\ y)\to(0,\ 0)} \frac{2xy^2}{x^2+y^4}$ does not exist.

(b) If $f(x, y) = \sqrt{|xy|}$, find $f_x(0, 0)$ and $f_y(0, 0)$.

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(c) Show that the function

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous at the origin.

2. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Examine the equality of $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ for the function

$$f(x, y) = \begin{cases} (x^2 + y^2)\tan^{-1}\left(\dfrac{y}{x}\right) & \text{if } x \neq 0 \\ \dfrac{\pi y^2}{2} & \text{if } x = 0 \end{cases}$$

5

(b) Show that the simultaneous limit and both the repeated limits for the following function exist at origin : 5

$$f(x, y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

(c) Show that the function

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & \text{if } x^4 + y^2 \neq 0 \\ 0 & \text{if } x = 0 = y \end{cases}$$

(Continued)

possesses first-order partial derivatives everywhere, whereas the function is discontinuous at the origin. 5

(d) State and prove the sequential criterion of limit for a function of two variables. 1+4=5

UNIT—2

3. Answer any *two* of the following questions : 2×2=4

(a) Find the gradient of the function $\log(x^2 + y^2 + z^2)$ at the point (1, 2, 1).

(b) Find the directional derivative of $f(x, y) = xe^y + \cos xy$ at the point (2, 0) in the direction of $\vec{u} = 3\hat{i} - 4\hat{j}$.

(c) Show that the function

$$f(x, y) = 2x^4 - 3x^2 y + y^2$$

has neither a maximum nor a minimum at the origin.

4. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) State and prove Young's theorem. 1+4=5

(b) Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 0,\ z = 0$. 5

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(c) State and prove Schwarz theorem. 5

(d) Find the maximum and minimum values of $x^2+y^2+z^2$ subject to $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$ and $z=x+y$. 5

UNIT—3

5. Answer any *two* of the following questions : 2×2=4

(a) Find the curl of

$$\vec{F}=(x^2-z)\hat{i}+xe^z\hat{j}+xy\hat{k}$$

(b) Define divergence of a vector field at a point. Find the divergence of

$$\vec{F}(x,y)=-\frac{y}{x^2+y^2}\hat{i}+\frac{x}{x^2+y^2}\hat{j}$$

at all the points in its domain.

(c) Evaluate

$$\iint(x^2+y^2)\,dxdy$$

over the rectangle [0, 1; 0, 2].

6. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Evaluate : 5

$$\int_0^3\int_1^{\sqrt{4-y}}(x+y)\,dxdy$$

(b) By changing to polar coordinates, show that

$$\iint_E\sqrt{x^2+y^2}\,dxdy=\frac{38\pi}{3}$$

where E is the region in the XY-plane bounded by $x^2+y^2=4$ and $x^2+y^2=9$. 5

(c) Evaluate : 5

$$\int_0^{\pi}\int_0^{\pi}|\cos(x+y)|\,dxdy$$

(d) Show that

$$\int_0^1 dx\int_0^1\frac{x-y}{(x+y)^3}\,dy\neq\int_0^1 dy\int_0^1\frac{x-y}{(x+y)^3}\,dx$$ 5

UNIT—4

7. Answer any *two* of the following questions : 2×2=4

(a) Evaluate

$$\int_0^1 dx\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}dy\int_0^a dz$$

by passing over to cylindrical coordinates.

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(b) Change the order of the following integration :

$$\int_0^1 dx \int_x^{\sqrt{x}} f(x, y)\, dy$$

(c) Evaluate the line integral $\int_C xy\, dx$, where C is the arc of the parabola $x = y^2$ from $(1, -1)$ to $(1, 1)$.

8. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Evaluate

$$\iiint_E xyz\, dxdydz$$

where E is the region bounded by $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$. 5

(b) Evaluate

$$\int_C \{(x - y)^3\, dx + (x - y)^3\, dy\}$$

where C is the circle $x^2 + y^2 = a^2$ in the anti-clockwise direction. 5

(c) Show that

$$\int_C \{(y^2 + z^2)dx + (z^2 + x^2)dy + (x^2 + y^2)dz\} = -2\pi ab^2$$

where the curve C is the part for which $z \geq 0$ of the intersection of the surfaces $x^2 + y^2 + z^2 = 2ax$, $x^2 + y^2 = 2bx$, $a > b > 0$, the curve begins at origin and runs at first in the positive octant. 5

(d) Find the volume cut from a sphere of radius a by a right circular cylinder with b as radius of the base and whose axis passes through the centre of the sphere. 5

UNIT—5

9. Answer any *two* of the following questions : 2×2=4

(a) Evaluate the line integral $\int_C (x^2 dx + xy\, dy)$ taken along the line segment from $(1, 0)$ to $(0, 1)$.

(b) State Stokes' theorem.

(c) State Greens' theorem in $\mathbb{R}^2$.

10. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Show that

$$\iint_S \{(y - z)\, dydz + (z - x)\, dzdx + (x - y)\, dxdy\} = \pi a^3$$

where S is the portion of the surface $x^2 + y^2 - 2ax + az = 0$, $z \geq 0$. 5

(b) Evaluate

$$\iint_S (x\, dydz + dzdx + xz^2\, dxdy)$$

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where S is the outer side of the part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. 5

(c) Use Stokes' theorem to find the line integral $\int_C (x^2y^3dx + dy + zdz)$, where C is the circle $x^2 + y^2 = a^2$, $z = 0$. 5

(d) Verify Green's theorem in the plane for $\oint_C \{(xy + y^2)dx + x^2dy\}$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 5

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