FYUG Odd Semester Exam., 2024

MATHEMATICS

(3rd Semester)

Course No. : MATSEC-201T

(Numerical Methods)

Full Marks : 50
Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *three* from the following : 1×3=3

(a) Define relative, absolute and round-off errors.

(b) Define truncation and percentage errors.

(c) Evaluate $\Delta^3(1-x)(1-2x^2)$.

(d) Define forward and backward difference operators.

Scanned with OKEN Scanner

2. Answer any *one* from the following : 2

(a) Prove that $\Delta\nabla = \Delta - \nabla = \delta^2$.

(b) Prove that $\mu = \left(1 + \frac{\delta^2}{4}\right)^{\frac{1}{2}}$.

3. *(a)* Round-off the numbers 865250 and 37·46235 to four significant digits and compute the absolute and relative errors in each case. 5

Or

(b) Prove that

$$hD = \log(1 + \Delta) = -\log(1 - \nabla) = \sinh^{-1}(\mu\delta)$$

UNIT—II

4. Answer any *three* from the following : 1×3=3

(a) Define interpolation.

(b) State Newton's forward interpolation formula.

(c) State Newton's backward interpolation formula.

(d) State Lagrange's interpolation formula.

5. Answer any *one* from the following : 2

(a) Construct a cubic polynomial which fits to the following data :

x	0	1	2	3
$f(x)$	1	2	1	10

(b) Obtain the Lagrange's interpolating polynomial for the following data :

x	1	3	4
y	0	6	12

6. *(a)* Find the value of y at $x = 2{\cdot}65$ from the following data : 5

x	−1	0	1	2	3
y	−21	6	15	12	3

Or

(b) Given that

$$\log_{10} 654 = 2{\cdot}8156$$
$$\log_{10} 658 = 2{\cdot}8182$$
$$\log_{10} 659 = 2{\cdot}8189$$
$$\log_{10} 661 = 2{\cdot}8202$$

Find the value of $\log_{10} 656$.

Scanned with OKEN Scanner

UNIT—III

7. Answer any *three* from the following : 1×3=3

(a) State the intermediate value theorem.

(b) Write the Newton-Raphson iterative formula.

(c) Write the sufficient conditions for the linear convergence of fixed-point iteration method.

(d) Discuss the geometrical significance of bisection method.

8. Answer any *one* question : 2

(a) Derive the Newton-Raphson formula to obtain $\sqrt[3]{N}$.

(b) Can we apply fixed-point iteration method to find the root of the equation $2x = \cos x + 3$ in $\left[0, \frac{\pi}{2}\right]$?

9. *(a)* Find the real root of the equation $\cos x = 3x - 1$ correct to four decimal places using fixed-point iteration method. 5

Or

(b) Find the positive real root of the equation $x^2 - \log_{10} x - 12 = 0$ by regula-falsi method.

UNIT—IV

10. Answer any *three* from the following : 1×3=3

(a) Write the general quadrature formula for numerical integration.

(b) Write the trapezoidal rule for a numerical integration.

(c) Write the Simpson's $\frac{1}{3}$rd formula for the approximation of $\int_a^b f(x)dx$.

(d) Write the error in the Simpson's $\frac{1}{3}$rd formula.

11. Answer any *one* from the following : 2

(a) Discuss the geometrical significance of Simpson's $\frac{3}{8}$th rule.

(b) Discuss the geometrical significance of trapezoidal rule.

12. *(a)* Find $y'(x)$ at $x = 0{\cdot}5$ for the following data : 5

x	0	1	2	3	4
$y(x)$	1	1	15	40	85

Scanned with OKEN Scanner

Or

(b) Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ by using trapezoidal rule, with $h = 0 \cdot 2$. Hence, determine the value of π.

UNIT—V

13. Answer any *three* from the following : 1×3=3

(a) Explain about pivoting.

(b) What is diagonally dominant matrix? Give an example.

(c) Write the sufficient conditions for the convergence of Gauss-Seidel method.

(d) Write the general form of the iteration formula for the Jacobi method.

14. Answer any *one* from the following : 2

(a) What are the advantages and disadvantages of the Gauss elimination method?

(b) Explain the importance of diagonal dominance in ensuring the convergence of the Jacobi method.

15. *(a)* Solve the following system of linear equations by Gauss-Jordan method : 5

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

Or

(b) Consider the system of equations

$$x - ay = b_1$$
$$-ax + y = b_2$$

where a is a real constant. For what values of a, Gauss-Jacobi and Gauss-Seidel methods will converge?

★ ★ ★

Scanned with OKEN Scanner