FYUG Even Semester Exam., 2026

MATHEMATICS

(4th Semester)

Paper Code : MTMDSC 253T

(Linear Algebra)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions.

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Check if the set of natural numbers is a vector space over $\mathbb{R}$ w.r.t. addition of natural numbers and multiplication of natural numbers by real numbers.

(b) Justify if the union of two subspaces is a subspace.

(c) Justify if the vectors $(1, 2, -1)$, $(3, 4, 7)$ and $(5, 8, 5)$ are linearly dependent.

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2. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Show that the set M_n of $n \times n$ matrices with real entries is a vector space over $\mathbb{R}$ w.r.t. suitably defined operations of vector addition and scalar multiplication. 5

(b) Let V be a vector space and $v, w \in V$. Show that

$$V_0 = \{\alpha v + \beta w | \alpha, \beta \in \mathbb{R}\}$$

is the smallest subspace of V containing v and w. 5

(c) Justify if the following are subspaces of $\mathbb{R}^3$ over $\mathbb{R}$: 6

(i) $V = \{(x, y, z) : x + 2y + 3z = 0\}$

(ii) $W = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$

(iii) $X = \{(x, y, z) : z = 0\}$

(d) Let V be a vector space. Show that a subset $\{v_1, v_2, \cdots, v_n\}$ of V is linearly dependent if and only if one of the vectors is a linear combination of the others. 4

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Justify if $S = \{(1, 0, 0), (0, 1, 0)\}$ is a basis of $\mathbb{R}^3$.

(b) Write a basis for the vector space $M_{2\times3}$ of all 2×3 matrices over $\mathbb{R}$ w.r.t. usual matrix addition and scalar multiplication.

(c) Let W be a subspace of V and $v \in V$. Show that if $v \in W$, then $v + W = W$.

4. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) If a vector space V has a basis of n elements, then show that any set of $n+1$ vectors is linearly dependent. 5

(b) Let W be a subspace of a vector space V. Show that the set V / W of all cosets of W in V is a vector space w.r.t. suitably defined operations. 5

(c) In a finite dimensional vector space, show that any two bases have the same number of elements. 5

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(d) Check if the following is a basis for $\mathbb{R}^3$:

$$S = \{v_1, v_2, v_3\}$$

where $v_1 = (1,2,3)$, $v_2 = (2,5,7)$ and $v_3 = (0,0,1)$. Also, if S is a basis, express any $(x, y, z) \in \mathbb{R}^3$ as a linear combination of v_1, v_2, v_3. 5

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Check if the map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x+y, y+z, z+3)$ is a linear transformation.

(b) If $T : V \to W$ be a one-one linear transformation, show that $\ker(T) = \{0\}$.

(c) Justify if there exists an isomorphism from $\mathbb{R}^5$ to $\mathbb{R}^3$.

6. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Let V and W be vector spaces. Let $\{v_1, v_2, \cdots, v_n\}$ be a basis of V. Let $w_1, w_2, \cdots, w_n$ be any n vectors in W. Then prove that there exists a unique linear transformation $T : V \to W$ such that $T(v_i) = w_i$ for all $i = 1, 2, \cdots, n$. 5

(b) Let $T : V \to W$ be a linear map. Show that $\ker(T)$ and Range(T) are subspaces of V and W respectively. 5

(c) Let $T : V \to W$ be a linear map, where V and W are finite dimensional. Show that T is an isomorphism if and only if for any basis $\{v_i : i = 1, 2, \cdots, n\}$ of V the set $\{T(v_i) : i = 1, 2, \cdots, n\}$ is also a basis of W. 5

(d) Find the matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(x, y, z) = (x+y, y+z, z+x)$$

w.r.t. the ordered basis

$$S = \{(1,1,0), (2,0,0), (0,1,2)\}$$ 5

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Justify if the set of all eigenvectors of a linear map $T : V \to V$ is a subspace of V.

(b) Write the eigenvectors of

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

corresponding to the eigenvalue -1.

(c) Justify if a non-singular matrix can have 0 as one of its eigenvalues.

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8. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Show that eigenvectors corresponding to distinct eigenvalues of a linear map $T : V \to V$ are linearly independent. 5

(b) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 0 & 0 & 3 \\ 0 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

and hence find A^{-1}. 5

(c) State and prove Cayley-Hamilton theorem. 5

(d) Find the eigenvectors and eigenvalues of

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

5

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Check if the following is an inner product on $\mathbb{R}^2$:

$$\langle (x_1, x_2), (y_1, y_2) \rangle = x_1 y_1 - x_2 y_2$$

(b) Show that in an inner product space V, $||x+y|| \leq ||x|| + ||y||$ for all $x, y \in V$, where $||x||^2 = \langle x, x \rangle$.

(c) If $x \perp y$ and $x \perp z$ in an inner product space, show that $x \perp (\alpha y + \beta z)$ for any scalars α and β.

10. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Show that $\langle z, w \rangle = \mathrm{Re}(z\bar{w})$ defines an inner product on the set $\mathbb{C}$ of complex numbers over $\mathbb{R}$. 5

(b) Let x, y be elements in an inner product space V. Show that—

(i) $||x+y|| = ||x|| + ||y||$ if and only if one is a non-negative scalar multiple of the other; 4

(ii) if x and y are orthogonal, then $||x+y||^2 = ||x||^2 + ||y||^2$. 1

(c) Show that any orthogonal set in an inner product space is linearly independent. 4

(d) Let W be a vector subspace of V. Then prove that $V = W \oplus W^{\perp}$, i.e., any $x \in V$ is uniquely written as $x = w + w'$ where $w \in W$ and $w' \in W^{\perp}$, i.e., the orthogonal complement of W. 6

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