FYUG Even Semester Exam., 2026

MATHEMATICS

(4th Semester)

Paper Code : MTMDSC-251T

(Abstract Algebra)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions.

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Construct a Cayley table for $G = \{1, \omega, \omega^2\}$. 2

(b) Give an example for each of the following : ½×4=2

(i) Finite Abelian group

(ii) Infinite Abelian group

(iii) Finite non-Abelian group

(iv) Infinite non-Abelian group

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(c) Define subgroup of a group. Is $\mathbb{R}^* \setminus \mathbb{Q}$ a subgroup of $\mathbb{R}^*$ under usual multiplication where $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$? 1+1=2

2. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Show that union of two subgroups is a subgroup if and only if one is contained in the other. 5

(b) Show that the set of all non-singular matrices forms a group under matrix multiplication. Is it Abelian? 4+1=5

(c) Show that a finite subset H of a group G is a subgroup if and only if it is non-empty and closed. 5

(d) Show that a non-empty subset H of a group G is a subgroup if and only if

$$ab^{-1} \in H \quad \forall a, b \in H$$ 5

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Show that every cyclic group is Abelian.

(b) Find all the generators of $\mathbb{Z}_{15}$.

(c) Find all the subgroups of $\mathbb{Z}_{30}$.

4. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Show that subgroup of a cyclic group is cyclic. 5

(b) Show that the center $Z(G)$ of a group G is a subgroup. 5

(c) Find the centralizer : 2½×2=5

(i) $C\left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right)$

(ii) $C\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right)$

(d) Let H be a subgroup of $\mathbb{R}$ under addition. Let $K = \{2^a \mid a \in H\}$. Prove that K is a subgroup of $\mathbb{R}^*$. 5

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Find even and odd permutation :

(i) (1 2)(1 3 4)(1 5 2)

(ii) (1 2 4 3)(3 5 2 1)

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(b) Find the order of permutation—

(i) (1 2 4)(3 5 7 8);

(ii) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 1 & 2 & 3 & 4 & 5 \end{bmatrix}$.

(c) Define 'permutation' and 'permutation group'.

6. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Show that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles. 5

(b) If the pair of cycles $\alpha = (a_1, a_2, \cdots, a_m)$ and $\beta = (b_1, b_2, \cdots, b_n)$ has no entries in common, then show that $\alpha\beta = \beta\alpha$. 5

(c) If a permutation α can be expressed as a product of even number of 2-cycles, then show that every decomposition of α into a product of 2-cycles must have an even number of 2-cycles. 5

(d) Define alternating group of degree n. Hence prove that alternating group of degree n has order $n!/2$. 5

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Show that A_n is a normal subgroup of S_n (symbols have usual meanings).

(b) Define quotient group.

(c) Every subgroup of an Abelian group is normal. Prove it.

8. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) State and prove Lagrange's theorem. 5

(b) If G is Abelian, then prove that G/N is Abelian where N is a normal subgroup of G. 5

(c) Show that a subgroup H of a group G is Abelian if and only if $Ha = aH$ for all $a \in G$. 5

(d) Let G be a group and $a \in G$. Then prove that—

(i) $O(a)$ divides $O(G)$;

(ii) $a^{O(G)} = e$, e is the identity element of G. 2½×2=5

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UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Define ring.

(b) Give an example of a ring which is not an integral domain and an integral domain which is not a field.

(c) Give an example for each of the following :

(i) Commutative ring

(ii) Non-commutative ring

10. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Prove that—

(i) $a0 = 0a = 0$;

(ii) $a(-b) = (-a)b = -(ab)$;

(iii) $(-a)(-b) = ab$;

(iv) $a(b-c) = ab - ac$;

(v) $(b-c)a = ba - ca$;

for all a, b and c belong to a ring R. 5

(b) Prove that a non-empty subset S of a ring R is a subring if $a-b$, $ab \in S$ for all $a, b \in S$. 5

(Continued)

(c) Show that

$$Z[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$$

is an integral domain. 5

(d) A finite integral domain is a field. Hence prove that $\mathbb{Z}_p$ is a field if p is a prime number. 5

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