FYUG Odd Semester Exam., 2023
(Held in 2024)

MATHEMATICS

(1st Semester)

Course No. : MATDSM–101T

(Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. State Cauchy's criterion for the existence of limit of a function.

2. Show that $\lim_{x\to 0} \cos\left(\frac{1}{x}\right)$ does not exist.

3. Show that if a function $f(x)$ is differentiable at a point $x = a$, then it is also continuous at $x = a$.

UNIT—II

4. Give the geometrical meaning of Rolle's theorem.

5. In the mean value theorem

$$f(x+h) = f(x) + hf'(x+\theta h)$$

If $f(x) = Ax^2 + Bx + C$, where $A \neq 0$, show that

$$\theta = \frac{1}{2}$$

6. Evaluate :

$$\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$$

UNIT—III

7. Define homogeneous function of degree n of two variables. Is the function $(\sqrt{x} + \sqrt{y} + \sqrt{z})$ is homogeneous? If so find its degree.

8. If $f(x, y) = e^{x^2 + xy + y^2}$, find f_{xx} and f_{xy}.

9. If $u = f\left(\frac{y}{x}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

UNIT—IV

10. If $f(x)$ is an even function, show that

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

11. If $f(x) = x - [x]$, then evaluate

$$\int_{-1}^{1} f(x)\,dx$$

where $[x]$ is the integral part of x.

12. Evaluate :

$$\int_{0}^{\pi/2} \sin^8 x \cos^6 x\, dx$$

UNIT—V

13. Find the length of arc of the parabola $y^2 = 16x$ measured from vertex to an extremity of the latus rectum.

14. What do you mean by rectification of plane curve? Write the formula to find the length of the curve $y = f(x)$ from $x = a$ to $x = b$.

15. Find the surface area of a solid generated by revolving the semicircular arc of radius c about the axis of x.

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 10×5=50

UNIT—I

16. *(a)* Using ε-δ definition of limit, evaluate

$$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$$ 5

(b) Examine the differentiability of the function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at $x = 0$. 5

17. *(a)* If $y = \sin(m \sin^{-1} x)$, then show that

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$$

Also find $y_n(0)$. 5

(b) State and prove Leibnitz's theorem on successive differentiation. 1+4=5

UNIT—II

18. *(a)* State and prove Lagrange's mean value theorem. 5

(b) Evaluate : 5

$$\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x}$$

19. *(a)* Show that the maximum value of $x^{1/x}$ is $e^{1/e}$. 5

(b) Write the statement of Maclaurin's theorem. Also expand $\sin x$ using Maclaurin's infinite expansion. 1+4=5

UNIT—III

20. *(a)* State and prove Euler's theorem on homogeneous function of degree n in two variables x and y. 5

(b) Find the tangent and normal to the curve

$$y(x-2)(x-3)-x+7=0$$

at the point where it cuts the x-axis. 5

21. (a) If

$$V=\log\left(\frac{x^3+y^3}{x^2+y^2}\right)$$

show that

$$x\frac{\partial V}{\partial x}+y\frac{\partial V}{\partial y}=1$$ 5

(b) Show that at any point on the curve

$$x^{m+n}=k^{m-n}y^{2n}$$

the mth power of the subtangent varies as the nth power of the subnormal. 5

UNIT—IV

22. (a) Obtain the reduction formula for

$$\int \sin^m x \cos^n x\, dx$$

where m, n are positive integers >1. 5

(b) Evaluate : 5

$$\int_0^{\pi/2}\left(\frac{1}{1+\cot x}\right)dx$$

23. (a) Obtain the reduction formula for

$$\int \sec^n x\, dx$$

n being a positive integer greater than 1. 5

(b) Prove that

$$\int_0^{\pi/2}\log(\sin x)\,dx=-\frac{\pi}{2}\log 2$$ 5

UNIT—V

24. (a) Find the area of the astroid

$$x^{2/3}+y^{2/3}=a^{2/3}$$ 5

(b) Find the area bounded by the parabola $y^2=4ax$ and its latus rectum. 5

25. (a) Find the surface area of the solid generated by revolving the cycloic

$$x=a(\theta+\sin\theta),\ y=a(1+\cos\theta)$$

about its base. 5

(b) The circle $x^2+y^2=a^2$ revolves around the axis. Find the surface area and the volume of the whole surface generated. 5

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