FYUG Odd Semester Exam., 2024

MATHEMATICS

(1st Semester)

Course No. : MATDSC-101T

(Higher Algebra and Trigonometry)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : $2\times2=4$

(a) State De Moivre's theorem.

(b) Write down the expansion of $\sin\alpha$, $\cos\alpha$, in terms of α.

(c) Find the equation whose roots are 7th powers of the roots of the equation $x^2-2x\cos\theta+1=0$.

2. *(a)* If $(1+x)^n = a_0 + a_1 x + a_2 x^2 + \cdots$ (n being a positive integer) then prove that—

(i) $a_0 - a_2 + a_4 - \cdots = 2^{\frac{n}{2}} \cos \frac{n\pi}{4}$;

(ii) $a_0 + a_4 + a_8 + \cdots = 2^{n-2} + 2^{\frac{n}{2}-1} \cos \frac{n\pi}{4}$.

3+4=7

(b) If $\frac{\sin \theta}{\theta} = \frac{2165}{2166}$, then show that θ is approximately 3°. 3

OR

3. *(a)* Prove that

$$\sin^2 \theta \cos \theta = \theta^2 - \frac{5}{6}\theta^4 + \cdots + (-1)^{n-1} \frac{3^{2n-1}}{4\lfloor 2n} \theta^{2n}$$ 4

(b) If $(a_1 + ib_1)(a_2 + ib_2) \cdots (a_n + ib_n) = A + iB$ then show that—

(i) $\tan^{-1} \frac{b_1}{a_1} + \tan^{-1} \frac{b_2}{a_2} + \cdots + \tan^{-1} \frac{b_n}{a_n} = \tan^{-1} \frac{B}{A}$;

(ii) $(a_1^2 + b_1^2)(a_2^2 + b_2^2) \cdots (a_n^2 + b_n^2) = A^2 + B^2$.

3+3=6

UNIT—II

4. Answer any *two* of the following questions : 2×2=4

(a) State Gregory's series.

(b) Separate $\sin(x+iy)$ into its real and imaginary part.

(c) Find the value of $\log(5+4i)$.

5. *(a)* Show that

$$\tan^{-1}(\cot\theta \tanh\phi) + \frac{1}{2i} \log \frac{\sin(\theta + i\phi)}{\sin(\theta - i\phi)}$$ 4

(b) Find the sum to infinity of

$$\cosh x + \frac{\sin x}{\lfloor 1} \cosh 2x + \frac{\sin^2 x}{\lfloor 2} \cosh 3x + \cdots$$ 4

(c) Write down the exponential values of $\sinh x$, $\cosh x$. 2

OR

6. *(a)* If $x < \sqrt{2} - 1$ then prove that

$$2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots\right) = \frac{2x}{1-x^2} - \frac{1}{3}\left(\frac{2x}{1-x^2}\right)^3 + \frac{1}{5}\left(\frac{2x}{1-x^2}\right)^5 - \cdots$$ 4

(b) If $x = \log \tan\left(\frac{\pi}{4} + \frac{y}{2}\right)$, then prove that

$$y = -i \log \tan\left(\frac{ix}{2} + \frac{\pi}{4}\right)$$ 4

(c) Find the value of $\sinh(x+iy)$. 2

Unit—III

7. Answer any *two* of the following questions : 2×2=4

(a) Define equivalence relation on a set *A*.

(b) Construct the truth table for the following compound statement :

$$p \to \sim (q \vee p)$$

(c) Fill in the blanks so that the resulting statement is equivalent to the implication $p \to q$:

(i) If _____ then _____.

(ii) _____ only if _____.

(iii) _____ is necessary for _____.

(iv) _____ is sufficient for _____.

8. *(a)* Prove that every equivalence relation on a set *S* determines a partition of *S*. 5

(b) Construct the truth table for the following compound statement : 5

$$(p \vee q) \leftrightarrow [((\sim p) \wedge p) \to (q \wedge p)]$$

OR

9. *(a)* If R_1 and R_2 be two equivalence relation on a set *A*, then prove that $R_1 \cap R_2$ is also an equivalence relation on *A*. Give one example to show that union of two equivalence relations is not an equivalence relation. 5

(b) *(i)* Write the inverse, converse and contrapositive of the following statement : 1+1+1=3

"If a triangle is equilateral, then it is isosceles."

(ii) Rewrite the following statements with universal and existential quantifiers : 1+1=2

(1) Every rational number is a real number.

(2) Not all birds can fly.

Unit—IV

10. Answer any *two* of the following questions : 2×2=4

(a) Write down Cauchy-Schwarz inequality and Minkowski inequality.

(b) Form the equation whose roots are the roots of the equation $3x^4 - 4x^3 + 13x - 7 = 0$ with sign changed.

(c) If α, β, γ be the roots of the equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$, find the value of $\Sigma \frac{1}{\alpha^2}$.

11. *(a)* If α, β, γ be the roots of the equation $x^3 - px^2 + qx - r = 0$, then form the equation whose roots are

$$\beta\gamma + \frac{1}{\alpha},\ \gamma\alpha + \frac{1}{\beta},\ \alpha\beta + \frac{1}{\gamma}$$ 4

(b) Solve the equation by Cardan's method $x^3 + 6x + 7 = 0$. 3

(c) Find the equation whose roots are square the roots of the equation $x^4 - 2x^3 + 3x^2 - x + 7 = 0$. 3

OR

12. *(a)* If *x*, *y*, *z* be positive rational numbers, prove that

$$\left(\frac{x^2 + y^2 + z^2}{x + y + z}\right)^{x+y+z} \geq x^x y^y z^z \geq \left(\frac{x + y + z}{3}\right)^{x+y+z}$$ 6

(b) Show that the equation

$$x^5 - x^4 + x^3 - 2x^2 - 3 = 0$$

has three positive roots and two imaginary roots. 4

UNIT—V

13. Answer any *two* of the following questions : 2×2=4

(a) What do you mean by elementary operation?

(b) If *A*, *B* are two square matrices of order *n* such that $AB = I$. Show that $BA = I$.

(c) Express the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

into a sum of symmetric and skewsymmetric matrix.

14. *(a)* Reduce the matrix

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

into normal form and hence find the rank of it. 5

(b) If

$$A = \begin{bmatrix} -2 & 1 & 3 \\ 0 & -1 & 1 \\ -2 & 2 & 1 \end{bmatrix}$$

show that *A* is non-singular. Find Adj *A* and A^{-1}. 5

OR

15. *(a)* What do you mean by Gaussian elimination method? Solve the equation

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

by Gaussian elimination method. 2+4=6

(b) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

by using elementary transformation. 4

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